# **Torsional Pendulum**

# EQUIPMENT

1	Torsion Pendulum Accessory	ME-6694	
1	Large Rod Stand	ME-8735	
1	60 cm Long Steel Rod	ME-8977	
1	Rotational Inertia Accessory	ME-3420	
1	Rotary Motion Sensor	PS-2120A	
1	High Resolution Force Sensor	PS-2189	11
	NOT INCLUDED, BUT REQUIRED	:	
1	Pliers for bending wire		and the
1	Mass Balance	SE-8757B	
1	Calipers	SE-8710	10
1	850 Universal Interface	UI-5000	
1	PASCO Capstone	UI-5400	

# INTRODUCTION

The torsional pendulum consists of a torsion wire attached to a Rotary Motion Sensor with an object (a disk, a ring, or a rod with point masses) mounted on top of it. The period of oscillation is measured from a plot of the angular displacement versus time. To calculate the theoretical period, the rotational inertia is determined by measuring the dimensions of the object and the torsional spring constant is determined from the slope of a plot of force versus angular displacement.

The dependence of the period on the torsional constant and the rotational inertia is explored by using different diameter wires and different shaped objects.



## THEORY

Consider a wire securely fixed on both ends. If the wire is twisted, it will exert a restoring torque when trying to return to its original untwisted position. For small twists, the restoring torque is proportional to the angular displacement of the wire.



 $\tau = \kappa \theta \qquad \tau = \kappa \theta \qquad (1)$ 

The proportionality constant,  $\kappa$ , depends on the properties of the wire and is called the torsional spring constant.

When the object attached to the wire is twisted and released, the object executes simple harmonic motion with a period, T, given by

$$T = 2\pi \sqrt{\frac{I}{\kappa}}$$
(2)

where I is the rotational inertia of the object about the axis of rotation.

Theoretically, the rotational inertia, I, of a ring is given by

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$
(3)

where M is the mass of the ring,  $R_1$  is the inner radius of the ring, and  $R_2$  is the outer radius of the ring. The rotational inertia of a disk is given by

$$I = \frac{1}{2}MR^2 \tag{4}$$

where M is the mass of the disk and R is the radius of the disk.

The rotational inertia of a point mass rotating in a circle of radius r is given by

$$I = MR^2 \tag{5}$$



#### SET UP

- 1. Start with the 0.032" diameter wire. Use pliers to bend each end of the wire into an "L" shape.
- 2. Fit the bent ends of the wire under the screws and washers of the upper and lower clamps, as illustrated in Figures 1 and 2. Make sure the screws are firmly tightened.



- 3. Adjust the Rotary Motion Sensor on the support rod such that the guide on the upper clamp is aligned with the slot on the shaft of the Rotary Motion Sensor. See Figure 3.
- 4. Adjust the height of the set up so that the upper clamp is approximately half way up the shaft of the Rotary Motion Sensor (see Figure 3). NOTE: When switching to a new diameter wire, try to keep the length of the wires, from clamp to clamp, relatively constant.
- 5. Plug the Rotary Motion Sensor and Force Sensor into the interface.



## PROCEDURE

## A. Determining the Torsional Spring Constant

- Measure the radius of the medium pulley of the Rotary Motion Sensor in meters. Enter this radius (not diameter!) into the Capstone calculator window where it asks for the experimental constants. The torque is calculated using τ=rF, where F is the force measured using the Force Sensor.
- 2. In PASCO Capstone, create a graph of torque vs. angle.
- 3. Attach about 20 cm of string to the Rotary Motion Sensor by tying it around the small pulley. Then thread the string through the notch in the medium pulley and wrap the string around the medium pulley 3 times. Attach the Force Sensor to the end of the string.
- 4. Set the sample rate for both sensors on 20 Hz.
- 5. Hold the force sensor parallel to the table at the height of the large pulley and prepare to pull it straight out as shown in Figure 4.



*Figure 4: Measuring the Torque* 

- 6. Let the string go slack and press the tare button on the Force Sensor. Click the RECORD button in PASCO Capstone and pull the Force Sensor horizontally until the pulley turns about one revolution. Click on STOP.
- 7. Use the Fit Tool to determine the slope of the graph of Torque vs. Angle. This slope is equal to the torsional spring constant for the wire (see Equation 1). Record the spring constant and the error in the spring constant.



#### **B.** Determining the Rotational Inertia

- 1. Measure the mass and radius of the disk.
- 2. Calculate the rotational inertia of the disk using Equation (4).

#### C. Calculating the Theoretical Period of Oscillation

Using the rotational inertia of the disk and the torsional spring constant for the wire, calculate the theoretical period using Equation (2). Use the error in the spring constant to estimate the error in the theoretical period.

#### **D.** Measuring the Period of Oscillation

- 1. Remove the Force Sensor. The string can still be attached in this part of the experiment as long as it does not impede the oscillation.
- 2. Change the sample rate to 200 Hz. Create a graph of the angle vs. time. Twist the disk 1/4 of a turn.
- 3. Start recording and release the disk.
- 4. After several oscillations have been completed, click on STOP.
- 5. Use the Coordinates Tool to find the period of oscillation. Measure the time of several periods and then divide by the number of periods.
- 6. Compare the measured and calculated values of the period using a percent difference.

$$\% difference = \frac{measured - calculated}{calculated} x100$$

## E. Repeating the Experiment

- 1. Repeat Steps B through D with the ring added to the top of the disk.
- 2. Remove the disk and the ring. Repeat Steps B through D using the rod with a point mass on each end of the rod. Invert the 3-step pulley on the Rotary Motion Sensor before attaching the rod (see Figure 5). For this part of the lab, the rotational inertia of the rod is ignored because it is small compared to the point masses. However, you can take the rotational inertia of the rod into account. For a thin rod of length L and mass m, the rotational inertia is  $I = \frac{1}{12}mL^2$ .





3. Replace the wire with a wire of different diameter but same length. Return to using the disk. Repeat Steps A, C, and D.

## QUESTIONS

- 1. Which of the wires was harder to twist? What does  $\kappa$  tell you about how much a wire resists bending and twisting?
- 2. Which of the wires oscillated faster (smaller Period)?
- 3. How does the period relate to which wire is harder to twist? Explain.
- 4. Using the same wire, which object had the least rotational inertia?
- 5. Using the same wire, which object oscillated faster?
- 6. How does the period depend on the rotational inertia of the object?
- 7. How much error is caused by ignoring the rod in the point mass part of the experiment?
- 8. Was there any other source of rotational inertia that was ignored in this experiment?
- 9. How could you use a torsional pendulum to determine the rotational inertia of any object that could be mounted on the Rotary Motion Sensor?

